Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 51 (2008) 1590–1595
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Free-convective heat transfer in fluids with non-uniform volumetric heat generation

P.S. Kondratenko, D.V. Nikolski *, V.F. Strizhov

Nuclear Safety Institute, Russian Academy of Sciences, 52, B. Tulskaya Street, 115191 Moscow, Russia

Received 3 April 2007; received in revised form 31 May 2007
Available online 27 September 2007

Abstract

The characteristics of natural convection of a heat-generating fluid with non-uniform distribution of volumetric heat release have been studied theoretically. The analysis was based on analytical estimates method and numerical simulation. It has been found that under particular conditions the details of volumetric heat release distribution over horizontal cross-sections of the liquid pool do not affect the convectional heat transfer characteristics. The vertical distribution of the horizontal cross-sectional mean value of volumetric heat generation completely determines the distribution of temperature in the bulk as well as the heat flux to the cooled boundary.

Keywords: NPP safety; Heat transfer; Heat-generating fluid; Free convection; Boundary layer

1. Introduction

The problem of NPP safety calls for investigations of heat transfer processes under severe accident conditions. In case of loss of coolant accident and the consequent core degradation it is possible for the heat-generating core melt to accumulate in the lower head of the reactor vessel. In such case the problem arises of keeping the integrity of the reactor vessel to prevent the outcome of radioactive materials. Currently the major strategies of the core melt retention for low and medium power reactors are external cooling and re-flooding of the core. The efficiency of external cooling is determined by the mechanism of water boiling on the external surface of the reactor vessel. To avoid the emergence of boiling crisis it is needed to know the local heat flux distribution, which is governed by natural convection of the heat-generating melt.

To validate the concept of the core melt retention in the lower head series of experimental studies were carried out most of them were conducted with model fluids such as water or salt melts and with prototype corium as well. The pools were heated by direct current, special heating wires installed in the pool, or by inductive heating (see [1] for example). In many cases the distribution of heat generation over the pool was not uniform as it is assumed for prototypic conditions. Particularly in case of inductive heating the heat is released mainly near the boundaries, the heating depth is governed by skin-effect and depends on the inductive current frequency. Thus the adequacy of heat transfer results should be justified to transfer results of investigations to prototype conditions with uniform distribution of heat release. This issue is what the present work is dedicated to. The study is carried out by means of the analytical estimates method (see [4,5]) and numerical simulation.

In Section 1 the problem statement and the physical model are formulated. Considered in this section is the case of thick heated layer (with respect to the free-convective boundary layer thickness). The opposite case is presented in Section 2. In Section 3 the applicability of the obtained results for more complex geometry is discussed, the effect of cooling of the upper horizontal boundary is considered as well. Section 4 is devoted to numerical simulation of free

* Corresponding author. Tel.: +7 495 955 2291; fax: +7 495 958 1151.
E-mail address: ndv@ibrae.ac.ru (D.V. Nikolski).

0017-9310/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2007.07.026

Nomenclature

BL	boundary layer
c	specific heat
g	acceleration due to gravity
H	pool height (fluid level)
P	pressure (hydrostatic component excluded)
$Pr = \nu/\chi$	Prandtl number
Q	volumetric heat release
$\overline{Q}$	cross-sectional average of Q
r	radial coordinate
R	pool radius
$Ra_I = \frac{g\alpha Q H^5}{\nu\chi\lambda}$	modified Rayleigh number
T	BL temperature counted from T_0
T_0	cooled boundary temperature
T_b	bulk temperature counted from T_0
u	longitudinal velocity component in the BL
$\vec{U}$	flow velocity in the bulk
U_z	vertical component of $\vec{U}$
U_r	radial component of $\vec{U}$
$\vec{v}$	flow velocity
v	transversal velocity component in the BL
y	coordinate normal to the lateral boundary
z	vertical coordinate

Greek symbols

α	thermal expansion coefficient
δ	free-convective boundary layer thickness near the lateral boundary
δ_Q	heated layer thickness
λ	thermal conductivity
ν	kinematic viscosity
ρ	density
$\chi = \frac{\lambda}{\rho c}$	thermal diffusivity

convection and heat transfer under the terms considered in Section 1 in order to validate the results obtained in that section.

1.1. Formulation of the problem: near-boundary heat-generating layer

The following analysis is performed for laminar flow. Stationary laminar natural convection of an incompressible fluid is described by the following set of equations (see [2,3]):

$$\mathrm{div}\,\vec{v} = 0, \tag{1}$$

$$(\vec{v}\nabla)\vec{v} = -\frac{\nabla P}{\rho} + \vec{g}\alpha T + \nu\Delta\vec{v}, \tag{2}$$

$$(\vec{v}\nabla)T = \chi\Delta T + \frac{Q}{\rho c_p}. \tag{3}$$

Here $\vec{v}$ is flow velocity, T – temperature, $Q = Q(\vec{r})$ – volumetric heat release, ν – kinematic viscosity, χ – thermal diffusivity, $\vec{g}$ – acceleration due to gravity, α – thermal expansion coefficient.

Let us assume that a fluid is confined in a vertical cylinder of radius R and height $H \sim R$. We suppose that the value of volumetric heat release does not depend on azimuthal coordinate

$$Q = Q(r, z), \tag{4}$$

where r is radial coordinate, z is vertical coordinate. We also suppose that only the lateral boundary is cooled and maintained at the constant temperature, the other boundaries are adiabatic. Temperature T will be counted from that of the cooled lateral boundary. General boundary conditions are discussed in Section 4.

Let us assume that heat is released in a layer adjacent to the lateral boundary, with the thickness δ_Q less than the dimensions of the cylinder R and H. First, we consider the case of the heated layer thickness δ_Q much more than the thickness of the free-convective boundary layer (BL) near the lateral boundary

$$\delta \ll \delta_Q. \tag{5}$$

Under condition (5), likely to [4,5], the heat generation inside the boundary layer can be neglected. Therefore the motion equations for the boundary layer can be written in the Prandtl approximation [3]

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$u\frac{\partial u}{\partial z} + v\frac{\partial u}{\partial y} - \nu\frac{\partial^2 u}{\partial y^2} = g\alpha(T - T_b), \tag{7}$$

$$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial y} - \chi\frac{\partial^2 T}{\partial y^2} = 0, \tag{8}$$

where u and v are longitudinal and transversal velocity components in the BL, T_b is the fluid temperature in the bulk (outside the boundary layers).

Besides Eqs. (6)–(8) the following estimates for the boundary layer are valid (see [4–7]):

$$v \sim \frac{\delta}{z}u, \tag{9}$$

$$\frac{u^2}{z} \sim g\alpha T_b, \tag{10}$$

$$\frac{u}{z} \sim \frac{\chi}{\delta^2}. \tag{11}$$

Here the derivation procedure is replaced with division by the characteristic spatial scales which are of order δ for radial coordinate and z for vertical coordinate, respectively. In the bulk (outside the BLs) the effects of thermal conductivity and viscosity are unimportant so the motion equations in this region take the following form (see [4–7]):

$$\operatorname{div}\vec{U} = 0, \tag{12}$$

$$(\vec{U}\nabla)\vec{U} = -\frac{\nabla P_{\mathrm{b}}}{\rho} + g\alpha T_{\mathrm{b}}\vec{n}, \tag{13}$$

$$\vec{U}\nabla T_{\mathrm{b}} = \frac{Q(r,z)}{\rho c}, \tag{14}$$

where $\vec{U}$ is the flow velocity in the bulk, ρ – density, P_{b} – pressure in the bulk minus hydrostatic component, c – specific heat, $\vec{n}$ – unit vector directed upwards.

According to the mass balance condition (1) the flow outside the boundary layer is much slower than inside the BL

$$U_z \sim \frac{\delta}{\delta_Q}u. \tag{15}$$

Here U_z is the vertical component of $\vec{U}$. In view of that fact that gravitational forces are almost balanced by pressure gradient – as in hydrostatics, all state functions of the fluid depend only on vertical coordinate. This gives a reason to propose that temperature stratification is set up in the bulk, in other words temperature in this region depends only on vertical coordinate [4–7]. This conclusion is confirmed by more accurate analysis below.

From the horizontal component of the momentum balance equation one can estimate the pressure variance order over horizontal cross-section at the certain z

$$(\Delta p)_{\mathrm{h}} \sim \rho U_r^2. \tag{16}$$

Through substitution of this expression into the vertical component of the momentum balance equation one can obtain the characteristic scale of the temperature variance at the same z

$$(\Delta T_{\mathrm{b}})_{\mathrm{h}} \sim \frac{U_r^2}{g\alpha z}. \tag{17}$$

Taking into account Eq. (9) and the fact that $U_r|_{r=R} = -v|_{\delta \ll y \ll \delta_Q}$ we can obtain the following expression for the radial velocity component in the bulk:

$$U_r \sim \frac{\delta}{z}u, \tag{18}$$

so we can express the characteristic temperature variance as follows:

$$(\Delta T_{\mathrm{b}})_h \sim \left(\frac{\delta}{z}\right)^2 T_{\mathrm{b}}. \tag{19}$$

Thus in the region of $z \gg \delta$ temperature stratification takes place, so that temperature is a function of vertical coordinate only, accurate to minor corrections of the order of (19)

$$T_{\mathrm{b}} \cong T_{\mathrm{b}}(z). \tag{20}$$

The energy balance equation can be written as follows:

$$U_z\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z} = \frac{Q(r,z)}{\rho c}. \tag{21}$$

Eq. (21) that describes the energy balance in the bulk is valid for any type of pool geometry. Geometrical factor can have an effect on the fitting conditions for the boundary layer and the bulk, which are boundary conditions for (21).

One has to notice that estimates (9)–(11) for the boundary layer at the lateral boundary, as well as Eq. (21) and estimates (16)–(19) for the bulk are not referred to the certain type of the heat release distribution. So they remain valid for arbitrary distribution, with one restriction: the heat generation inside the boundary layer should be negligible with respect to the rest of the volume.

Another noticeable consequence of (21) is that vertical velocity component U_z depends on radial coordinate just as the heat generation rate

$$U_z = \frac{Q(r,z)}{\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z}\rho c}. \tag{22}$$

The mass balance (12) can be written as

$$\frac{\partial U_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(U_r \cdot r) = 0. \tag{23}$$

Through averaging (23) over the horizontal cross-section one can obtain the following expression for the radial velocity component at the BL periphery:

$$U_r(R) = -\frac{R}{2}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z}}\right)\frac{\overline{Q}(z)}{\rho c}. \tag{24}$$

As we see, the radial velocity component is determined only by the cross-sectional average value of internal heat release

$$\overline{Q}(z) = \frac{1}{\pi R^2}\int_0^R Q(r,z)\cdot 2\pi r\cdot \mathrm{d}r \tag{25}$$

and does not depend on details of its distribution over the radial coordinate r.

Eq. (21) can be integrated over the horizontal cross-section with regard to the mass balance condition

$$2\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z}\int_0^{y>\delta} u\cdot \mathrm{d}y = -R\frac{\overline{Q}(z)}{\rho c}. \tag{26}$$

One should notice that relations (24) and (26) define the correspondence between the BL and the bulk characteristics, thus closing the set of Eqs. (6)–(8) and (21).

Whether the value of $\overline{Q}$ does not depend on z the closing relations (24) and (26) are similar to those for the case of uniform heat generation. Then the temperature distribution in the bulk and consequently the boundary heat flux distribution will also correspond to the uniform heat generation case.

In view of the stated conclusions inequality (5) becomes the condition under which the correspondence between the non-uniform (near-boundary) and the uniform heat generation cases can be reached. To specify this condition it is necessary to determine the parameter δ (the characteristic thickness of the free-convectional boundary layer at the lateral boundary). The thickness of the boundary layer at the sidewall of a vertical cylindrical pool with uniform heat generation has been estimated in papers [6,7]. It is determined by modified Rayleigh number Ra_I, and for laminar flow regime, which corresponds to $Ra_I < 10^{12}$, its order is

$$\delta \sim R \cdot Ra_I^{-1/5}.$$

In the problem considered with non-uniform heat generation it is convenient to express the modified Rayleigh number through the volume average of the heat generation rate $\overline{Q}_v$

$$Ra_I = \frac{g\alpha\overline{Q}_v H^5}{v\chi\lambda},$$

where λ is thermal conductivity. The inequality (5) thus takes the form

$$\frac{\delta_Q}{R} \gg Ra_I^{-1/5}. \quad (27)$$

2. Thin heat-generating layer

Let us concern the case of axisymmetrical distribution of heat release, as described in Section 1, but with the heating depth δ_Q much less than the thickness of hypothetical free convectional boundary layer at the lateral boundary δ

$$\delta \gg \delta_Q. \quad (28)$$

In this case, the heat is generated in a thin layer near the boundary where the role of convective transport is insufficient with respect to molecular heat conduction. So practically all the heat released is removed through the boundary by means of the latter. The temperature at the heated layer periphery (see Eq. (3)) is of order

$$T(z) \sim \delta_{\rm q}^2 \frac{\overline{Q}(z)}{\lambda}. \quad (29)$$

The natural convection pattern will coincide with that for a fluid without internal heat generation, the effective boundary conditions given by (29). In particular, if both the upper and the lower boundaries are adiabatic there will be no reason for convection to be established.

3. Hemispherical geometry: cooled upper boundary

The above analysis is applicable to more complex axisymmetrical geometry, e.g. hemispherical. Then, under condition (5), the balance equations for the bulk remain as above; the boundary layer equations and the closing relations as well are written with regard to the geometrical factor [4,5]. Valid for them are estimates analogous to (9)–(11). The conditions of temperature stratification occurrence in the bulk are the same as found above. The energy balance outside the BLs is described, as noticed, by Eq. (21). Correspondingly, the stratification regime is determined by distribution of cross-sectional average of volumetric heat release. Particularly, whether this value is independent on the vertical coordinate the temperature distribution in the stratified region should be the same as if the heat generation was uniform.

The parameter δ in condition (5) is now defined, according to [4], as

$$\delta \sim R \cdot Ra_I^{-1/6}.$$

So the analogy to the condition of possible correspondence (27) for hemispherical geometry is

$$\frac{\delta_Q}{R} \gg Ra_I^{-1/6}. \quad (30)$$

So far, the boundary condition at the upper horizontal boundary was assumed to be adiabatic. In case of cooled upper boundary the maximal temperature is achieved inside the pool. Thereby in the region above the temperature maximum level the conditions for Rayleigh–Benard convection are established. In the range of Rayleigh numbers interesting for the practical applications ($10^8 < Ra_I < 10^{15}$) turbulent convection regime is set up in this region. Owing to this the time–average temperature of the core (outside the boundary layers) is practically uniform. So the heat flux distribution to the boundary of the Rayleigh–Benard region must be specified by the averaged over this region volumetric heat release.

4. Numerical experiment

By means of the FLUENT 6.2 code [8] a series of calculations of free convection in a cylindrical pool was carried out. The heat release distribution was assumed either uniform, or given by the expression

$$Q(r) = \frac{\overline{Q}}{A}\mathrm{ch}\left(\frac{r}{\delta_Q}\right), \quad (31)$$

where A is given by the following expression:

$$A = \delta_Q \mathrm{sh}\left(\frac{1}{\delta_Q}\right) - \delta_Q^2\left(\mathrm{ch}\left(\frac{1}{\delta_Q}\right) - 1\right),$$

where r is the radial coordinate, the value of $\overline{Q}$ corresponds to volumetric heat release in case of uniform heat generation. The temperature of the lateral and the bottom boundaries was assumed constant, while the upper boundary was adiabatic. Presented below are the results of calculations with $Ra_I = 10^9$, $R = H$, and the heated layer thickness $\delta_Q = 0.2 \cdot R$, that is deliberately larger than the boundary layer thickness at the given Rayleigh number. Figs. 1–4 show temperature and vertical velocity distributions, streamlines in the bulk and velocity profiles at the half-height horizontal cross section of the pool. The left side on the figures corresponds to the boundary, the right side corresponds to the axis.

It is well seen on the graphs that the region of temperature stratification occupies the whole volume except the boundary layers, including the region without heat generation, right to the axis. Vertical flow velocity outside the boundary layer quickly decreases with the distance from

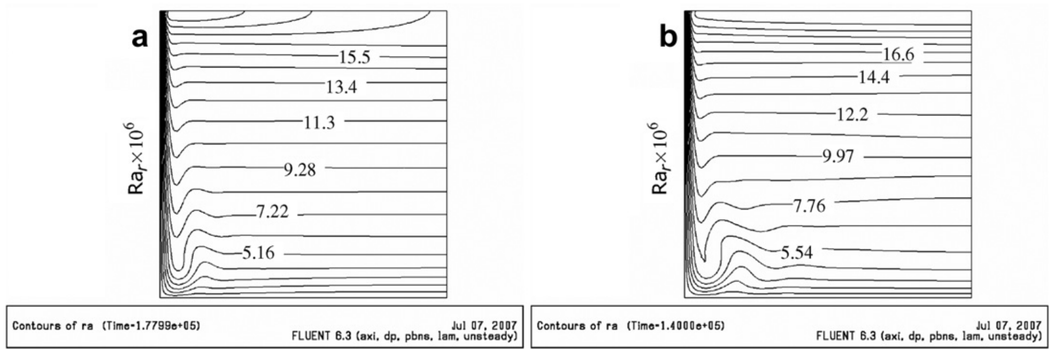

Fig. 1. Distribution of local Rayleigh number $\left(Ra_r = \frac{g\alpha T(r,z)H^3}{\nu\chi}\right)$ in cases of non-uniform (a) and uniform (b) heat generation.

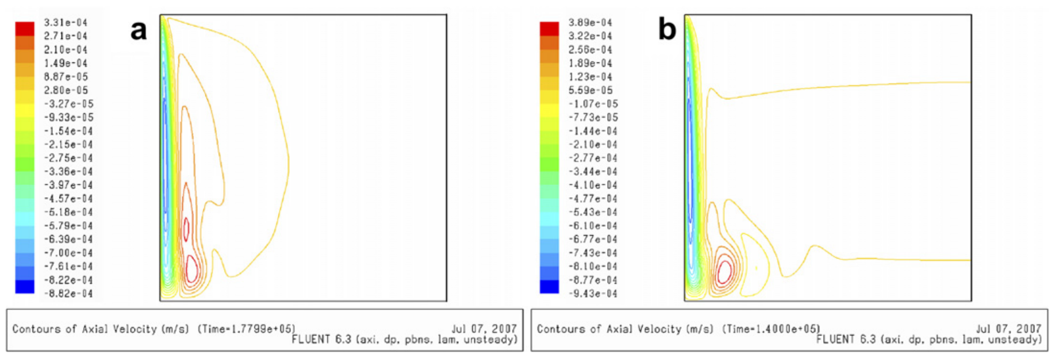

Fig. 2. Vertical velocity distribution in cases of non-uniform (a) and uniform (b) heat generation.

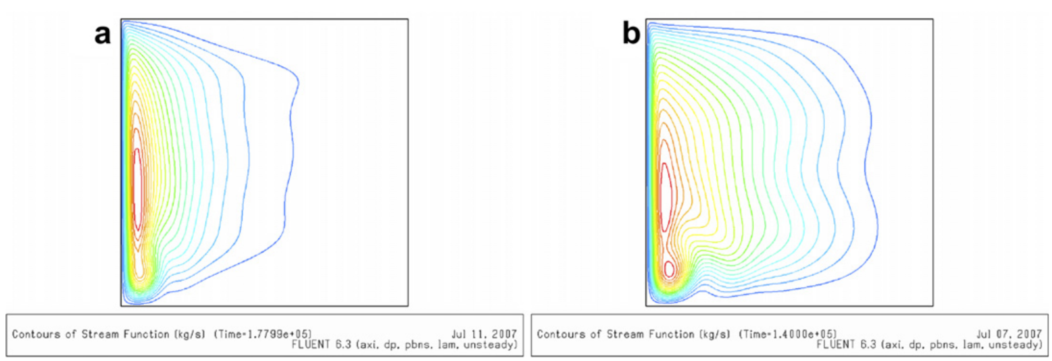

Fig. 3. Streamlines in cases of non-uniform (a) and uniform (b) heat generation.

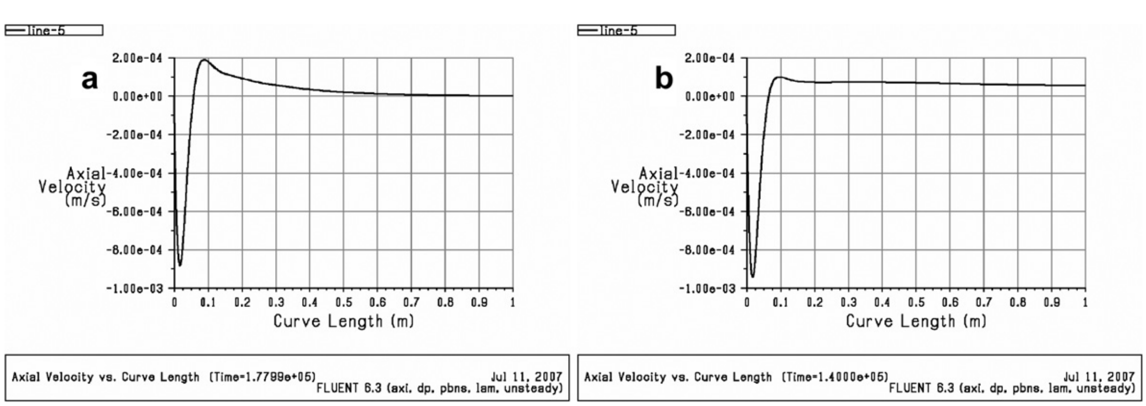

Fig. 4. Vertical velocity *vs* transversal coordinate y at $z = 0.5 \cdot H$ in cases of non-uniform (a) and uniform (b) heat generation.

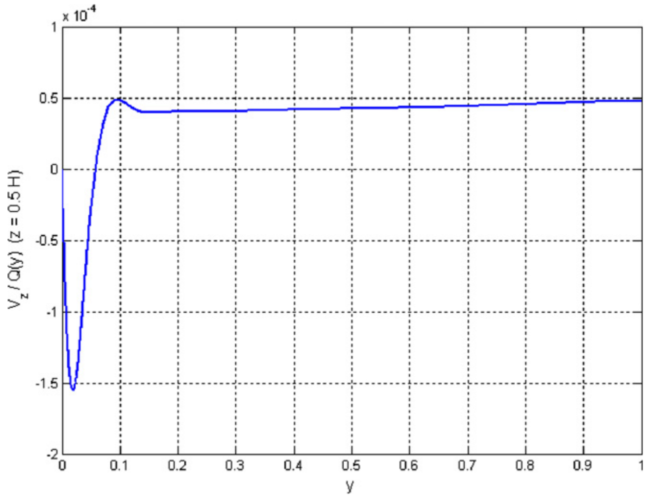

Fig. 5. The ratio of vertical velocity to the local volumetric heat release rate (dimensionless) *vs* transversal coordinate at $z = 0.5 \cdot H$ (in case of non-uniform heat generation).

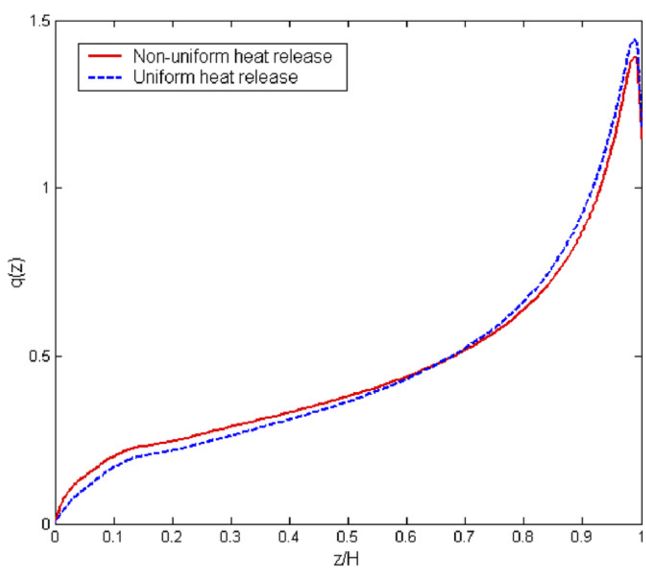

Fig. 6. Boundary heat flux distribution in cases of non-uniform (solid curve) and uniform (dashed curve) heat generation.

the boundary, in the same manner as the heat release rate does (see Fig. 5). This agrees well with the theoretical results obtained above. Fig. 6 shows close correspondence of the sidewall heat flux distribution to that in case of uniform heat generation.

5. Conclusions

The main results of the work are as follows:

Under condition of the smallness of the boundary layer thickness, as compared to the characteristic scale of volumetric heat release inhomogeneity, the general structure of free convection (the stable stratification region, the Rayleigh–Benard layer near the cooled upper boundary, free-convective boundary layers) is the same as with uniform heat generation (see [4,6]). In such case, many details of heat release distribution play the minor role; the key factor is the vertical (axial) distribution of the heat generation (average volumetric heat release over the horizontal cross-section) $\overline{Q}(z)$. In case of $\overline{Q}(z) = \text{const}$ entire quantitative correspondence to the uniform heat generation case is achieved.

If the heat-generating layer thickness is small with respect to the hydrodynamic boundary layer thickness the flow structure corresponds to free convection without internal heat sources. The effective boundary temperature distribution in this case is governed by the boundary heat release.

Acknowledgement

We acknowledge support from RFBR under Project 05-08-17964.

References

[1] F.J. Asfia, B. Frantz, V.K. Dhir, Experimental investigation of natural convection heat transfer in volumetrically hested spherical segments, ASME J. Heat Transfer 118 (1996) 31–37.
[2] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1987.
[3] B. Gebhart et al., Buoyancy-induced Flows and Transport, Hemisphere Publishing, New York, 1988.
[4] L.A. Bolshov, P.S. Kondratenko, V.F. Strizhov, Natural convection in heat-generating fluids, Phys.-Usp. 44 (10) (2001) 999–1016.
[5] L.A. Bolshov, P.S. Kondratenko, Limiting angular dependencies of heat and mass transfer in a heat generating fluid, Int. J. Heat Mass Transfer 43 (2000) 3897–3905.
[6] D.G. Grigoruk, P.S. Kondratenko, Free convection of a heat generating fluid in cylindrical geometry, in: Proceedings of the Third Russian National Conference on Heat Transfer (RNKT-3), Moscow, vol. 3, 2002, pp. 57–60 (in Russian).
[7] D.G. Grigoruk, P.S. Kondratenko, D.V. Nikolski, Geometrical factor in free convection of heat generating fluids, in: Izvestiya RAN, Energetika, vol. 2, 2004, pp. 86–100 (in Russian).
[8] Fluent Inc., Fluent 6.2 User's Guide, Lebanon, 2005.